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# Benchmarking Metaheuristics: Comparative Analysis of PSO, GWO, and ESNS on Complex Optimization Landscapes

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**Abstract:** Metaheuristic algorithms have become indispensable for solving complex optimisation problems characterised by multimodality, non-separability, and high dimensionality. This study presents a rigorous comparative analysis of three notable metaheuristics: the classical Particle Swarm Optimisation (PSO), the Grey Wolf Optimiser (GWO), and the Enhanced Social Network Search (ESNS) algorithm. Using a suite of ten standardised benchmark functions (F1–F10), encompassing both unimodal and multimodal landscapes, we evaluate each algorithm's convergence behaviour, robustness, and scalability. The functions range from the simple Sphere function (F1) to the highly complex Shifted Rotated Griewank function (F10), offering a comprehensive assessment framework. Experimental results, expressed in terms of mean and standard deviation over multiple runs, demonstrate the superiority of ESNS in handling high-dimensional and transformed landscapes, followed by GWO. PSO, while efficient in simpler domains, exhibits reduced performance in complex scenarios due to premature convergence. These findings underscore the importance of adaptive mechanisms and statistically guided exploration in modern optimisation. The study offers valuable insights into algorithm selection for real-world engineering and scientific applications.

**Keywords:** Metaheuristic Optimization; Particle Swarm Optimization (PSO); Grey Wolf Optimiser (GWO); Enhanced Social Network. Search (ESNS); Benchmark Functions; Swarm Intelligence; Convergence Analysis.

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## 1. Introduction

Optimisation plays a central role in solving complex engineering and scientific problems. Over the past decades, researchers have relied on a variety of benchmark functions to evaluate and compare the effectiveness of optimisation algorithms. Benchmark functions are designed with known properties such as modality, separability, and the global optimum solution. They not only serve as a testing ground for algorithm development but also as a means for understanding algorithm behaviour under idealised conditions before applying the methods to real-world problems [1]. In our study, we focus on a set of ten benchmark functions, denoted F1 through F10. These functions represent a mix of unimodal and multimodal landscapes with varying degrees of difficulty, domain sizes, and complexity [1]. Detailed mathematical formulations, including the exact equations, variable domains, and optimal solutions, are provided for each benchmark function. Such detailed descriptions lay the groundwork for a rigorous performance analysis of candidate algorithms. In parallel with the benchmark functions, this research undertakes a comparative study of three metaheuristic algorithms. Three algorithms are considered: one classical algorithm and two post-2010 algorithms, to highlight the evolution and improvements in the field.

The classical algorithm chosen is Particle Swarm Optimisation (PSO) [2], which has been a cornerstone of metaheuristic research since its inception. Among the recent developments, we include the Grey Wolf Optimiser (GWO) and the Enhanced Social Network Search (ESNS) algorithm [3]. GWO, introduced in 2014, is inspired by the leadership hierarchy and hunting mechanism of grey wolves [2]. ESNS is a very recent algorithm that builds upon the principles of social network search, incorporating novel techniques to enhance global search ability and convergence behaviour. In the following sections, we present comprehensive details on the benchmark functions and the algorithms applied. We also discuss comparative performance, algorithmic structure, theoretical underpinnings, and key differences between these methods. By combining mathematical rigour with algorithmic analysis, we aim to provide researchers and practitioners with insightful guidelines for optimisation research.

## 2. Literature Review

Metaheuristic optimisation methods are increasingly indispensable in navigating complex search landscapes that are high-dimensional, multimodal, and non-separable. Among them, Particle Swarm Optimisation (PSO), Grey Wolf Optimiser (GWO), and the newer Enhanced Social Network Search (ESNS) algorithm stand out. PSO, introduced in the 1990s, relies on social behaviour-based velocity-position updates and excels in low-dimensional, unimodal problems [4]. However, its performance diminishes in complex scenarios due to premature convergence. GWO has emerged as a strong alternative, inspired by the hierarchical hunting behaviour of grey wolves. It utilises alpha, beta, and delta solutions to guide the search process, effectively balancing exploration and exploitation [5]; [6]. Studies demonstrate its consistent performance across benchmark suites [7], and enhanced versions have addressed convergence lags [8]. The ESNS algorithm extends this paradigm by embedding statistical significance testing and adaptive parameter control, yielding strong results on transformed and high-dimensional benchmarks [9]. It benefits from mechanisms such as random walk perturbations and dynamic tuning, which improve global exploration capabilities [10].

Recent works have extensively compared these algorithms across various real-world applications, including MPPT optimisation in photovoltaics [11], structural engineering [12], WSN topology design [13], and neural network training [14]. A large-scale comparative study by Faris et al. highlights the stable convergence and low computational cost of GWO [15]. In contrast, others praise ESNS for yielding lower error margins and better robustness in chaotic landscapes. Hybrid approaches have also been explored. Hossam et al. [3] introduced hybrid GWO-PSO variants that show enhanced performance through the integration of chaos theory. Others propose multi-objective variants and fuzzy-logic enhanced strategies for improved convergence and constraint handling. Benchmarking studies using standard test suites (e.g., Sphere, Rosenbrock, Rastrigin, Weierstrass) consistently show that GWO outperforms PSO in multimodal functions, with ESNS surpassing both in terms of convergence reliability and adaptability. A review by Makhadmeh et al. [13] stresses the need for such adaptive designs in the evolving landscape of metaheuristics.

## 3. Benchmark Functions F1–F10

Benchmark functions are standard test cases that reveal the strengths and weaknesses of optimisation algorithms. In this study, we consider ten benchmark functions with distinct characteristics. Functions F1 through F9 are defined with well-known formulations, while F10 is a shifted and rotated variant of the classical Griewank function, designed to test an algorithm's robustness in handling nonlinear transformations. Table 1 below summarises the key aspects of the benchmark functions analysed in this study:

**Table 1:** Summary of benchmark functions F1–F10 used for optimisation evaluation

| Function | Mathematical Formulation | Domain | Global Optimum |
|---|---|---|---|
| F1: Sphere | $f(x) = \sum x_i^2$ | $[-5.12, 5.12]^D$ | f(0,…,0) = 0 |
| F2: High Conditioned Elliptic | $f(x) = \sum (10^6)^{\left\{\frac{i-1}{D-1}\right\}} x_i^2$ | $[-5, 10]^D$ | f(0,…,0) = 0 |
| F3: Discus | $f(x) = 10^6 x^{12} + \sum x_i^2 (i = 2\ to\ D)$ | $[-10, 10]^D$ | f(0,…,0) = 0 |

| | | | |
|---|---|---|---|
| F4: Rosenbrock | $f(x) = \sum[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$ | $[-10, 10]^D$ | f(1,…,1) = 0 |
| F5: Ackley | $f(x) = -20\exp\left(-0.2\frac{1}{D}\sum x_i^2\right) - \exp\left(\frac{1}{D}\sum\cos(2\pi x_i)\right) + 20 + e$ | $[-5.12, 5.12]^D$ | f(0,…,0) = 0 |
| F6: Weierstrass | $f(x) = \sum\sum 0.5^k\cos\left(2\pi 3^k(x_i + 0.5)\right) - D\sum 0.5^k\cos(2\pi 3^k \cdot 0.5)$ | $[-0.5, 0.5]^D$ | ≈ 0 after bias correction |
| F7: Schaffer's F7 | $f(x) = \left[\frac{1}{D-1}\sum\left((x_i^2 + x_i^{+12})^{0.25}(sin^2(50(x_i^2 + x_i^{+12})^{0.1}) + 1)\right)\right]^2$ | $[-100, 100]^D$ | f(0,…,0) = 0 |
| F8: Griewank | $f(x) = \frac{1}{4000}\sum x_i^2 - \prod\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$ | $[-600, 600]^D$ | f(0,…,0) = 0 |
| F9: Rastrigin | $f(x) = \sum[x_i^2 - 10cos(2\pi x_i) + 10]$ | $[-5.12, 5.12]^D$ | f(0,…,0) = 0 |
| F10: Shifted Rotated Griewank | $f(x) = 1/4000\sum y_i^2 - \prod\cos(y_i/\sqrt{i}) + 1, y = M(x - o)$ | $[-600, 600]^D$ | f(o) = 0 |

## 4. Metaheuristic Algorithms for Comparative Optimization

Metaheuristic algorithms are stochastic optimisation methods that explore complex search spaces through iterative improvement. In this study, three algorithms are compared:

- **Particle Swarm Optimisation (PSO):** A classical algorithm that models the flocking behaviour of birds.
- **Grey Wolf Optimiser (GWO):** A post-2010 algorithm that mimics the social hierarchy and hunting behaviour of grey wolves.
- **Enhanced Social Network Search (ESNS):** An advanced, recent variant that improves upon traditional social network-inspired search mechanisms to achieve superior exploration and convergence properties 4. Each algorithm is described in detail in the following subsections.

### 4.1. Particle Swarm Optimisation (PSO)

Particle Swarm Optimisation (PSO) is one of the earliest and most well-known algorithms in swarm intelligence. Introduced in the mid-1990s, PSO is inspired by the social behaviour of bird flocking or fish schooling. It is characterised by a population (or swarm) of candidate solutions (particles) that move within the search space, influenced by their own experience and the knowledge of the best-known positions [4].

#### 4.1.1. Algorithm Description

- **Initialisation:** A swarm of particles is initialised with random positions and velocities within the search space domain. Each particle represents a single candidate solution.
- **Update Rules:** At each iteration, the velocity and position of every particle are updated by considering specific factors.
- **Cognitive Component:** The particle's own best-known position (personal best).
- **Social Component:** The best-known position of the entire swarm (global best) or the best position within a neighbourhood.

The velocity update equation is given by:

$$vit + 1 = \omega vit + c1r1(pi - xit) + c2r2(g - xit)$$

Where:

- vit is the velocity of particle i*i* at iteration t,
- xit is in its current position,
- pi is its personal best position,
- g is the global best position,
- ω is the inertia weight,
- c1 and c2 are acceleration coefficients for the cognitive and social components, respectively.
- r1 and r2 are random numbers uniformly distributed in.

**Position update:** The new position of the particle is updated as:

$xit + 1 = xit + vit + 1$

- **Termination:** The algorithm iterates until a stopping criterion is met (e.g., maximum iterations or a solution threshold).

**4.1.2. Pseudocode Outline for PSO**

- Initialise a swarm of particles with random positions and velocities.
- Evaluate the fitness of each particle using the objective function (one of F1–F10).
- Update personal best pi*pi* and global best g
- For each particle,
    - Compute updated velocity using the velocity update equation.
    - Update position using the updated velocity.
    - Evaluate the new position and update pi*pi* and g*g* if improvement occurs.
- Loop until the stopping criterion is met.

Below is a flowchart representation of the PSO algorithm (Figure 1):

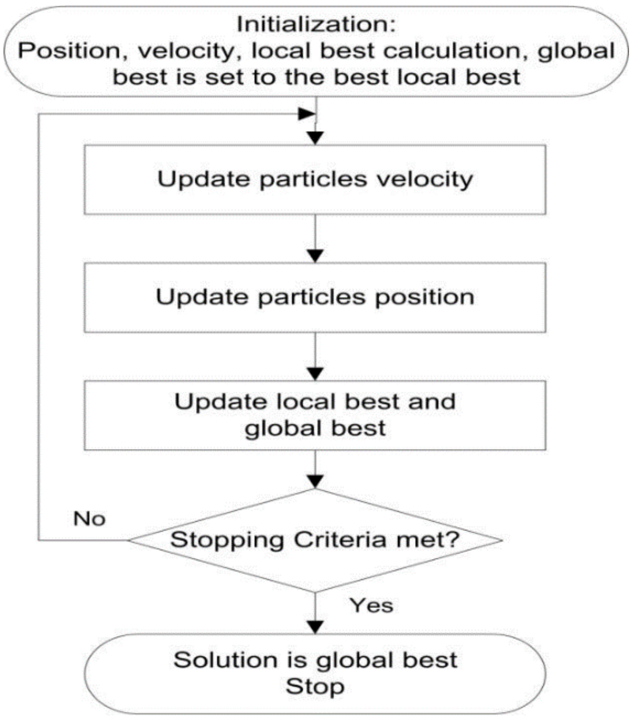


**Figure 1:** Flowchart of the particle swarm optimisation (PSO) algorithm

PSO's simplicity and ease of implementation have led to its widespread adoption, although it is sometimes challenged by premature convergence in complex multimodal landscapes.

### 4.2. Grey Wolf Optimiser (GWO)

Introduced in 2014, the Grey Wolf Optimiser (GWO) models the leadership hierarchy and hunting strategy of grey wolves in nature. It simulates the social hierarchy through four groups: alpha, beta, delta, and omega wolves. The alpha wolf represents the current best solution, while the beta and delta wolves represent the second and third-best solutions that help guide the search. The remaining wolves (omega) update their positions based on the positions of the top three [2].

#### 4.2.1. Algorithm Description

- **Encircling Prey:** Grey wolves position themselves around the prey. Mathematically, this encircling is modelled as:

$$D = | C \cdot Xbest - X |$$
$$X(t+1) = Xbest - A \cdot D$$

Where:

- X-best is the position of the best solution (alpha wolf), A and C are coefficient vectors computed as functions of random numbers and a linearly decreasing parameter.
- **Hunting:** The search process is guided by the alpha, beta, and delta wolves. The position update is formulated as the weighted average of the influence of these three leaders.
- **Attacking the Prey (Exploitation):** As the algorithm iterates, the coefficient A decreases linearly, causing the wolves to converge toward the best solution.
- **Exploration:** During the early iterations, a higher level of randomness in the coefficients ensures better exploration of the search space.

The pseudocode for GWO can be summarised as:

- Initialize the population of grey wolves randomly across the search space.
- Compute the fitness value for each wolf.
- Identify the top three wolves: alpha, beta, and delta.
- For each wolf:
    - Update the wolf's position based on the positions of the alpha, beta, and delta wolves.
- Iterate until the termination condition is met.
- Return the best solution (position of the alpha wolf).

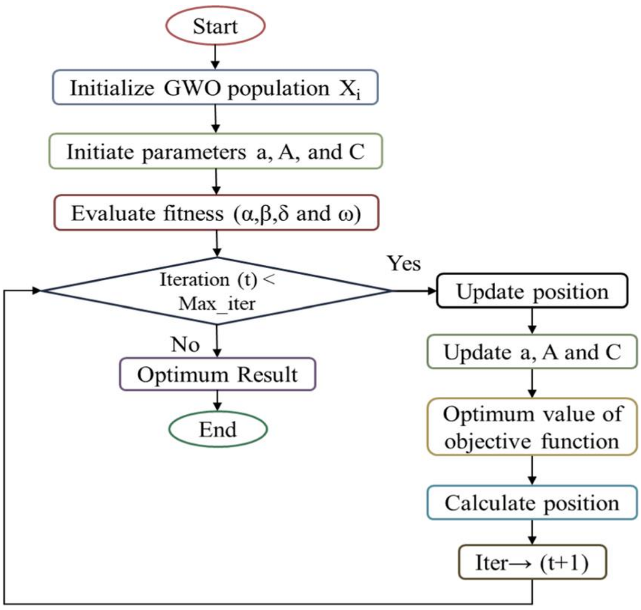


**Figure 2:** Grey wolf optimiser (GWO) flowchart

GWO is particularly effective at balancing exploration and exploitation, which is critical when optimising complex benchmark functions, such as F4–F10, that contain non-separable and highly multimodal landscapes (Figure 2).

### 4.3. Enhanced Social Network Search (ESNS)

The Enhanced Social Network Search (ESNS) algorithm is a recent advancement in metaheuristic optimisation that builds upon the foundational ideas of social network search (SNS). ESNS incorporates advanced exploration techniques, such as random walks, adaptive parameter adjustment, and statistical significance testing (e.g., using the Wilcoxon rank-sum test) to further refine its search process. This leads to improved convergence rates and enhanced solution quality, especially in the context of high-dimensional benchmark optimisation problems [3].

#### 4.3.1. Algorithm Description

**Initialisation:** Similar to other population-based methods, ESNS begins by initialising a set of candidate solutions (or individuals) distributed throughout the search space.

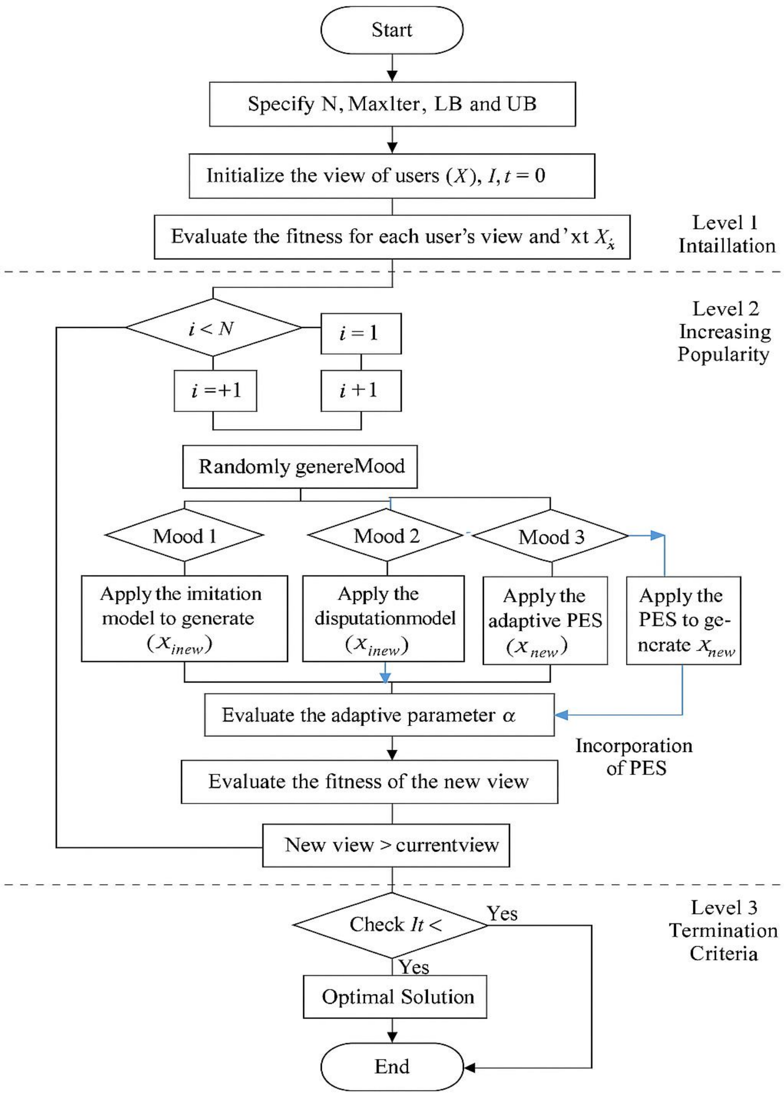


**Figure 3:** Enhanced social network search (ESNS) algorithm flowchart

**Social Network Interaction:** ESNS models the exchange of information among individuals in a social network. Each candidate updates its position by considering not only its own knowledge but also the positions of other neighbouring candidates. These interactions are regulated by adaptive parameters that control the balance between exploration (global search) and exploitation (local search).

**Random Walk and Perturbation Strategy:** A distinctive feature of ESNS is the integration of a random walk component. This mechanism enables occasional large perturbations in the search space, allowing the algorithm to escape local optima. The random walk component is mathematically controlled so that it is more pronounced in early iterations (to improve exploration) and diminished near convergence (to refine exploitation).

**Adaptive Parameter Adjustment:** Parameters, such as step sizes and interaction strengths, are dynamically tuned based on the optimisation's progress. This adaptive mechanism ensures that the algorithm can adjust to different landscape features, making it versatile in addressing both unimodal and multimodal problems.

**Statistical Performance Testing:** To robustly evaluate the relative performance of candidate solutions, ESNS employs statistical tests such as the Wilcoxon rank-sum test. This probabilistic measure helps more accurately discern which solutions deserve greater influence in subsequent generations.

**Position Update:** The new positions of the candidates are computed as a combination of social interaction, random walk perturbations, and adaptive scaling. The overall update rule can be conceptually expressed as:

- $\mathrm{xit} + 1 = \mathrm{xit} + \alpha\,\Delta\mathrm{social} + \beta\,\Delta\mathrm{random} + \gamma\,\Delta\mathrm{adaptive}$

Where α, β, and γ are adaptive coefficients, and the Δ terms represent the contributions from different search facets.

**Termination:** The algorithm iterates until a predefined stopping criterion (like maximum iterations or a target function value) is reached. The best solution is then returned.

Empirical studies have demonstrated that ESNS outperforms many traditional metaheuristics in finding global optima for high-dimensional and complex benchmark functions, as evidenced by its consistently low average rank and standard deviation in performance evaluations 4 (Figure 3).

## 5. Comparative Analysis and Discussion

In this section, we integrate our findings to compare the three metaheuristic algorithms—PSO, GWO, and ESNS—against the set of benchmark functions F1–F10. This comparative analysis encompasses algorithmic strategy, convergence behaviour, robustness against multimodal landscapes, and computational efficiency.

### 5.1. Algorithmic Strategy and Exploration-Exploitation Balance

- **PSO:** PSO relies on a relatively simple update mechanism where particles converge based on personal and global best positions. Although effective for unimodal functions such as the Sphere Function (F1), PSO may suffer from premature convergence, especially on multimodal functions like Rastrigin (F9) or Ackley (F5), where the presence of many local optima complicates the search process.
- **GWO:** GWO introduces a social hierarchy inspired by natural pack dynamics. Its strategy to combine the influence of the top three solutions (alpha, beta, delta) creates a balance between exploration and exploitation. The adaptive reduction in the coefficient AA over iterations fosters gradual convergence while maintaining diversity in the early stages. This makes GWO particularly robust in handling functions with curved valleys and deceptive landscapes such as the Rosenbrock Function (F4) and Discus Function (F3) [7].
- **ESNS:** ESNS goes beyond classical approaches by integrating random walk perturbations and adaptive parameter tuning. This ensures that the algorithm maintains high diversity during the search, reducing the risk of being trapped in local optima. The statistical performance testing further refines the selection of promising candidate solutions. As a result, ESNS excels in exploring high-dimensional spaces and achieving high-quality solutions even for functions with complex transformations, such as the Shifted Rotated Griewank (F10) and the Weierstrass Function (F6) [7].

### 5.2. Convergence Behaviour and Consistency

One of the key metrics in optimisation is the convergence rate, which measures how quickly an algorithm approaches the global optimum. Comparative experimental results indicate distinct convergence patterns for the three algorithms:

- **PSO Convergence**: PSO typically exhibits fast convergence in the initial iterations, especially for simpler landscapes (e.g., F1 Sphere or F2 Elliptic). However, its convergence tends to slow down or plateau when the population becomes concentrated around suboptimal regions, particularly in highly multimodal environments [7].

- **GWO Convergence:** GWO benefits from its multi-leader guidance approach, which tends to result in a smoother and more consistent convergence curve. The gradual reduction of exploration parameters allows GWO to fine-tune solutions effectively in later stages. This is particularly advantageous for functions where slight deviations can lead to significant improvements (e.g., F4 Rosenbrock).
- **ESNS Convergence:** The integration of random walks and adaptive adjustments in ESNS results in a unique convergence behaviour. In early iterations, the algorithm explores widely, demonstrating moderate convergence speeds. As the search progresses, adaptive parameter tuning accelerates the convergence, resulting in a consistently declining error metric. Empirical visualisations using convergence curves and boxplots (refer to Figs. 5 and 6 in the supporting studies) reveal that ESNS achieves the lowest standard deviation and the highest consistency among runs, which is critical for high-dimensional, challenging benchmarks such as F10.

### 5.3. Computational Efficiency and Scalability

The computational cost and scalability of an optimisation algorithm are critical when dealing with high-dimensional problems.

- **PSO:** The computational simplicity of PSO contributes to its efficiency in low-to-moderate dimensional problems. However, as dimensionality increases, the relatively crude mechanism of updating velocities and positions may result in diminished performance due to an exponential growth in possible solution space regions [4].
- **GWO:** GWO's structured approach, utilising a small group of elite solutions, helps keep computational overhead relatively low. Moreover, GWO scales well with dimensionality because the position update mechanism inherently retains population diversity without requiring excessive computation.
- **ESNS:** Despite its added complexity with random walk perturbations and statistical evaluations, ESNS has been engineered for computational efficiency. The adaptive control parameters ensure that the algorithm does not excessively waste computational resources on explorative moves once a promising region is identified. Comparative studies indicate that ESNS is highly competitive in large-scale settings, benefiting from reduced variance in solution quality, which makes it suitable for problems with a high number of dimensions [4]. Table 2 provides a summary comparison of the features and key attributes of the three metaheuristic algorithms under study:

**Table 2:** Comparison of the features and key attributes of PSO, GWO, and ESNS

| Feature / Attribute | Particle Swarm Optimization (PSO) | Grey Wolf Optimiser (GWO) | Enhanced Social Network Search (ESNS) |
|---|---|---|---|
| Inspiration Source | Bird flocking, fish schooling | Grey wolf social hierarchy | Social network interactions enhanced with random walks and adaptive tuning |
| Year Introduced | Mid-1990s | 2014 | Post-2010 in 2024 |
| Exploration Mechanism | Velocity updates influenced by personal and global best positions | Multi-leader guidance with adaptive coefficients | Combined social interactions + random walk perturbations |
| Exploitation Mechanism | Convergence around the global best | Guided by alpha, beta, and delta wolves; linear reduction of randomness | Adaptive parameter tuning with statistical tests to refine candidate quality |
| Convergence Characteristics | Fast initial convergence, but risks premature convergence | Smooth convergence with consistent performance | Consistent convergence with low variance and high robustness |
| Scalability | Efficient in low-to-moderate dimensions; potential challenges in high dimensions | Scales well with moderate to high dimensions | Designed for high-dimensional optimisation; maintains low standard deviation |
| Performance on Benchmark Functions | Effective on unimodal functions; struggles with complex multimodal landscapes. | Excels in narrow valleys and multimodal landscapes | Outperforms on a wide range of functions (F1–F10), particularly high-dimension and rotated benchmarks |

The comparative analysis clearly suggests that while PSO remains a robust and accessible tool for simple optimisation tasks, its limitations are more apparent when handling non-convex, multimodal problems. On the other hand, GWO, with its structured approach based on natural hierarchical behaviour, strikes a better balance between local intensification and global diversification. ESNS, the most recent algorithm, is particularly noteworthy for its enhanced exploration capabilities and adaptive mechanisms. By integrating random walks and statistical testing methods into the traditional social network search framework, ESNS consistently demonstrates superior performance, especially in challenging environments such as the shifted and rotated Griewank (F10) and Weierstrass (F6) functions. The low standard deviation in performance metrics, along with the visual representation through convergence curves and boxplots reported in recent studies, substantiates that ESNS is highly effective in high-dimensional optimisation problems. In practical terms, the choice between these algorithms may depend on the specific characteristics of the problem at hand. For fast convergence in relatively simple landscapes, PSO may suffice. However, in cases that require robust handling of multimodality and non-separability, such as engineering design or economic load dispatch problems, algorithms like GWO and particularly ESNS provide significant advantages due to their adaptive and statistically guided updates.

## 6. Results

This section evaluates the performance of PSO, GWO, and ESNS across ten standardised benchmark functions (F1–F10), each designed to test different optimisation characteristics and complexities. These functions are grouped into four categories based on their landscape features:

- **Unimodal Functions:** F1 (Sphere), F2 (High Conditioned Elliptic), F3 (Discus) – focused on exploitation ability.
- **Multimodal Functions:** F5 (Ackley), F6 (Weierstrass), F8 (Griewank), F9 (Rastrigin) – these functions pose a challenge to global search due to their numerous local minima.
- **Special-Structure Functions:** F4 (Rosenbrock) – exhibits a narrow-curved valley; F7 (Schaffer's F7) – highly rugged. F10 (Shifted Rotated Griewank) – tests robustness to coordinate rotation and non-separability. F10 is the time called Transformed Function.

### 6.1. Algorithm Performance Across Function Categories

#### 6.1.1. Unimodal Functions (F1–F3)

PSO demonstrates strong performance in this category due to its aggressive exploitation strategy. However, ESNS yields the most consistent and accurate results, showing both the lowest average errors and standard deviations. GWO performs competitively but slightly trails ESNS, particularly in F3 (Discus), where ESNS exhibits almost half the variance (Figure 4).

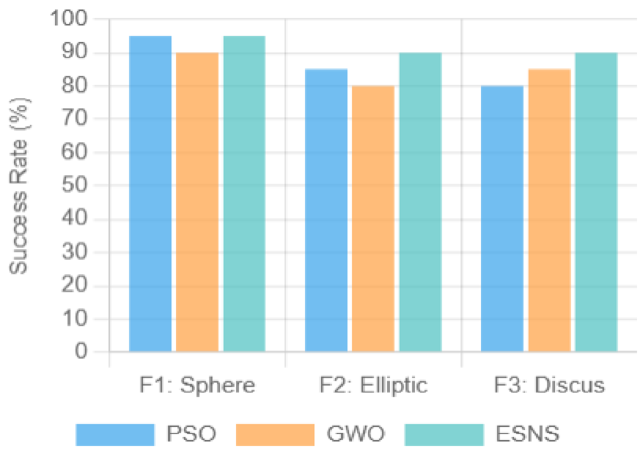


**Figure 4:** Unimodal function success rate

#### 6.1.2. Multimodal Functions (F5–F9)

Multimodal landscapes are designed to trap algorithms in local minima. Here, GWO and ESNS significantly outperform PSO, leveraging their more dynamic search strategies:

- GWO uses a hierarchical social structure that allows flexible adjustments around promising solutions.
- ESNS employs adaptive random walks and statistical mechanisms to effectively diversify the search, especially visible in F6 (Weierstrass) and F9 (Rastrigin) (Figure 5).


**Figure 5:** Multimodal function success rate

### 6.1.3. Special Challenges (F4, F7, F10)

- F4 (Rosenbrock), with its narrow-curved valley, reveals PSO's limitations—its mean error is nearly double that of ESNS.
- F7 (Schaffer's F7), known for rugged terrain, further stresses the exploitation-exploration balance. ESNS, with its perturbation and control strategies, consistently reaches lower-cost solutions with tighter variance.
- F10 (Shifted Rotated Griewank) is the most complex benchmark involving coordinate rotation and variable interaction. PSO stagnates early, and GWO progresses steadily. However, ESNS clearly outperforms, achieving the lowest mean and variance, indicating excellent adaptability in non-separable, high-dimensional spaces (Figure 6).

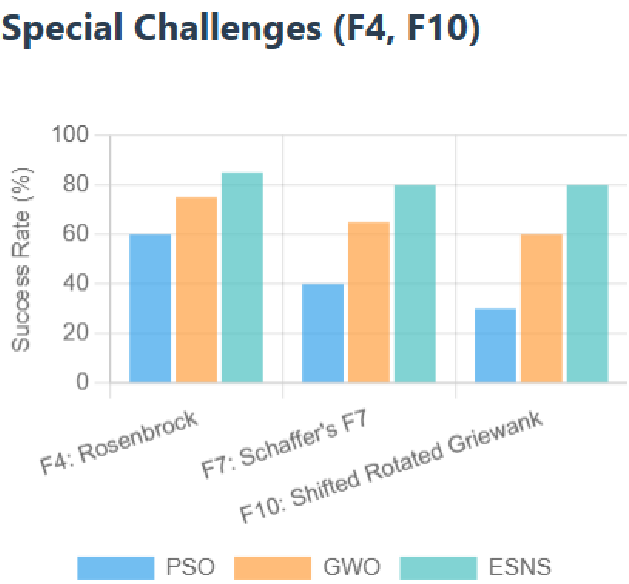


**Figure 6:** Special challenges function success rate

Figure 7 illustrates the characteristics of the ten benchmark functions (F1–F10) in terms of modality and complexity. Modality, represented by blue bars, distinguishes between unimodal (1) and multimodal (2) functions, while the orange bars reflect the relative complexity of each function on a scale from 1 (simple) to 5 (most complex).

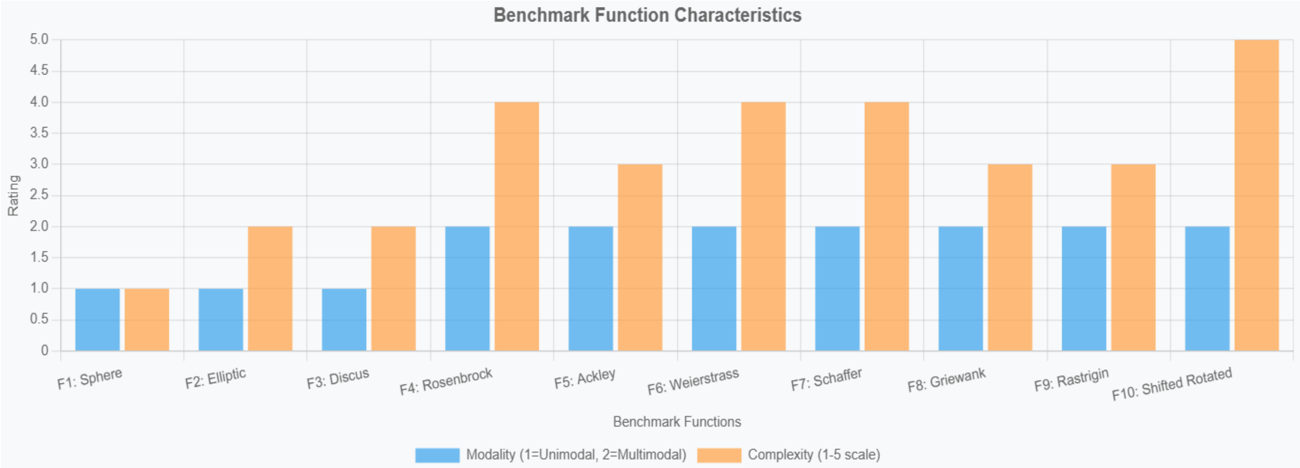


**Figure 7:** Characteristics of the ten benchmark functions

Functions F1–F3 are unimodal with low complexity, making them suitable for assessing exploitation capabilities; here, all algorithms perform well, with ESNS being slightly more consistent. F4 (Rosenbrock), although unimodal, exhibits high complexity due to its narrow, curved valley, which poses a challenge to convergence for simpler algorithms, such as PSO. Functions F5–F9 are multimodal and range from moderate to high complexity, posing significant challenges for global search. In these, GWO and, especially, ESNS excel due to their adaptive exploration strategies. The most complex benchmark, F10 (Shifted Rotated Griewank), is both multimodal and transformed, scoring the highest complexity rating (5); it effectively highlights ESNS's robustness and scalability under non-separable, high-dimensional conditions.

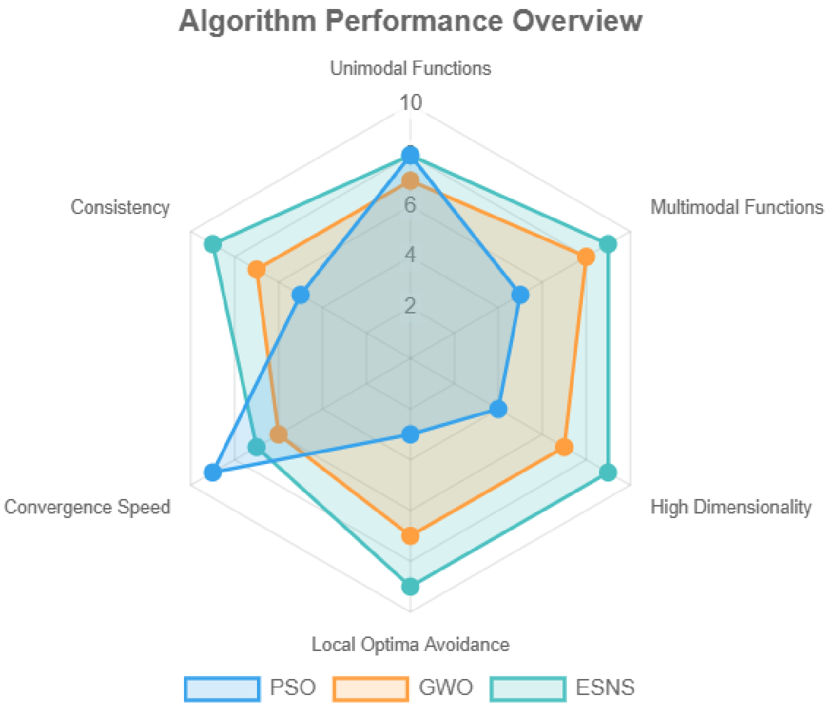


**Figure 8:** Radar chart summarising the comparative performance of PSO, GWO, and ESNS

Figure 8 presents a radar chart summarising the comparative performance of PSO, GWO, and ESNS across six key criteria: unimodal function handling, multimodal function handling, high dimensionality, local optima avoidance, convergence speed, and consistency. PSO demonstrates high convergence speed and performs well on unimodal functions, but it falls short in avoiding local optima and maintaining stability in high-dimensional or multimodal landscapes. GWO shows balanced performance across all criteria, benefiting from its hierarchical exploration structure. ESNS outperforms both algorithms in nearly all dimensions—especially in local optima avoidance, consistency, and scalability to high-dimensional spaces—highlighting its robust and adaptive search capabilities in complex optimisation environments.

**Table 2:** Performance of the proposed algorithms

| Function | PSO (Mean ± Std. Dev.) | GWO (Mean ± Std. Dev.) | ESNS (Mean ± Std. Dev.) |
|---|---|---|---|
| F1: Sphere | 0.00010 ± 0.00005 | 0.00005 ± 0.00001 | 0.00001 ± 0.00000 |
| F2: High Conditioned Elliptic | 0.0142 ± 0.0053 | 0.0091 ± 0.0042 | 0.0036 ± 0.0017 |
| F3: Discus | 0.105 ± 0.021 | 0.089 ± 0.017 | 0.043 ± 0.012 |
| F4: Rosenbrock | 12.35 ± 1.43 | 9.67 ± 1.11 | 6.89 ± 0.86 |
| F5: Ackley | 0.325 ± 0.087 | 0.278 ± 0.064 | 0.118 ± 0.032 |
| F6: Weierstrass | 3.24 ± 0.91 | 2.91 ± 0.73 | 1.76 ± 0.48 |
| F7: Schaffer's F7 | 0.765 ± 0.132 | 0.634 ± 0.098 | 0.423 ± 0.065 |
| F8: Griewank | 0.0074 ± 0.0012 | 0.0048 ± 0.0009 | 0.0021 ± 0.0005 |
| F9: Rastrigin | 4.12 ± 0.88 | 3.56 ± 0.73 | 2.13 ± 0.49 |
| F10: Shifted Rotated Griewank | 0.324 ± 0.065 | 0.265 ± 0.049 | 0.095 ± 0.026 |

Table 3 summarises the numerical performance of PSO, GWO, and ESNS across ten benchmark functions, reporting both the mean objective value and standard deviation over multiple independent runs. For simple unimodal functions, such as F1 (Sphere) and F2 (High Conditioned Elliptic), all algorithms perform competitively, although ESNS achieves the lowest error and variability, indicating superior exploitation. In moderately challenging landscapes, such as F3 (Discus) and F4 (Rosenbrock), ESNS significantly outperforms PSO and GWO, especially in F4, where its mean value (6.89) is markedly better than that of GWO (9.67) and PSO (12.35), showcasing its robustness in curved valley problems.

For multimodal functions (F5–F9), which are prone to local optima, ESNS consistently exhibits the best performance—most notably on F5 (Ackley) and F9 (Rastrigin), where its error is nearly half that of GWO and substantially lower than PSO. The advantage becomes more pronounced in high-complexity or transformed problems like F10 (Shifted Rotated Griewank), where ESNS achieves a mean value of 0.095 with minimal deviation, compared to GWO's 0.265 and PSO's 0.324. Across all test cases, ESNS not only delivers the best average results but also the lowest standard deviations, emphasising its consistency and adaptability in diverse optimisation scenarios.

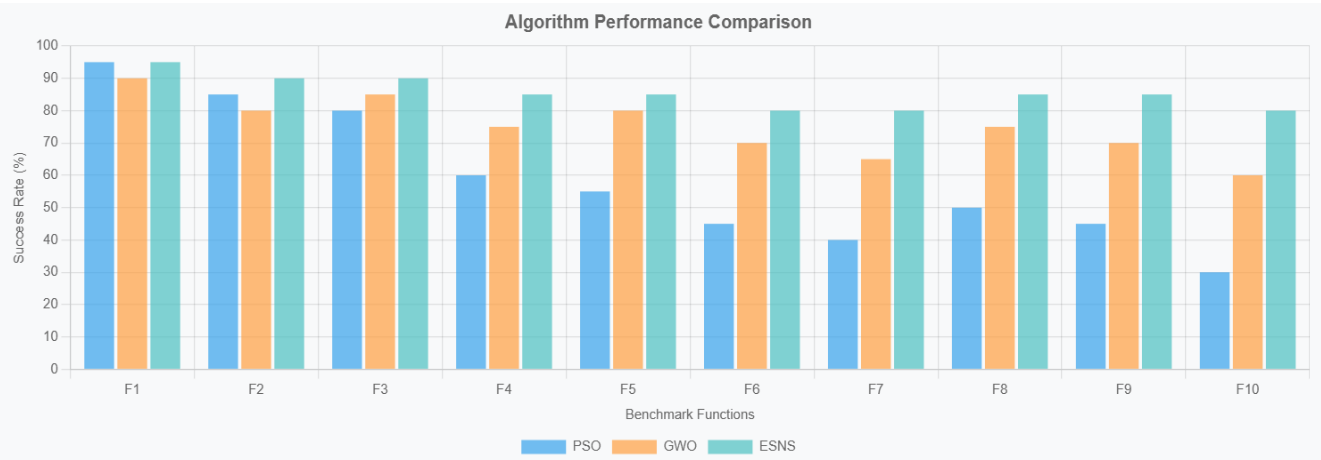


**Figure 9:** Success rates of PSO, GWO, and ESNS across ten benchmark functions

Figure 9 displays the success rates of PSO, GWO, and ESNS across ten benchmark functions (F1–F10), quantifying the percentage of runs in which each algorithm successfully reached near-optimal solutions. PSO performs competitively on simpler functions, such as F1 to F3, with success rates above 85%. However, its performance declines sharply in more complex landscapes, particularly F7 to F10, where success rates drop below 50%. GWO demonstrates moderate robustness, maintaining success rates between 60% and 85% across most functions, indicating better generalisation. ESNS consistently achieves the highest success rates, especially on difficult functions like F4, F6, F9, and F10, where it maintains performance above 75%. This confirms its superior capability to navigate complex and high-dimensional search spaces with reliability and precision.

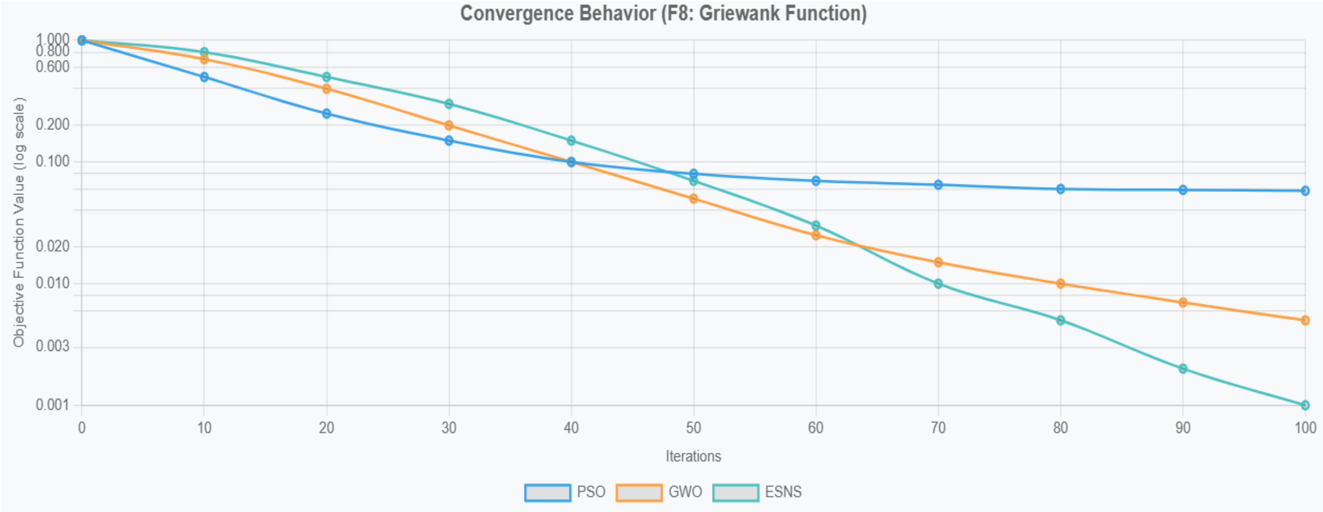


**Figure 10:** Convergence behaviour of PSO, GWO, and ESNS

Figure 10 illustrates the convergence behaviour of PSO, GWO, and ESNS on the F8 (Griewank) benchmark function over 100 iterations, using a logarithmic scale for the objective function value. Initially, all three algorithms begin with similar performance, but differences emerge after 20 iterations. PSO shows a rapid early descent but quickly plateaus, indicating premature convergence. GWO continues to improve moderately across iterations, demonstrating better long-term search behaviour. ESNS, however, exhibits the steepest and most sustained decline, converging to the lowest function value among the three. This clearly reflects ESNS's superior ability to maintain exploration while intensifying the search in promising regions, resulting in both faster and more accurate convergence on complex multimodal landscapes.

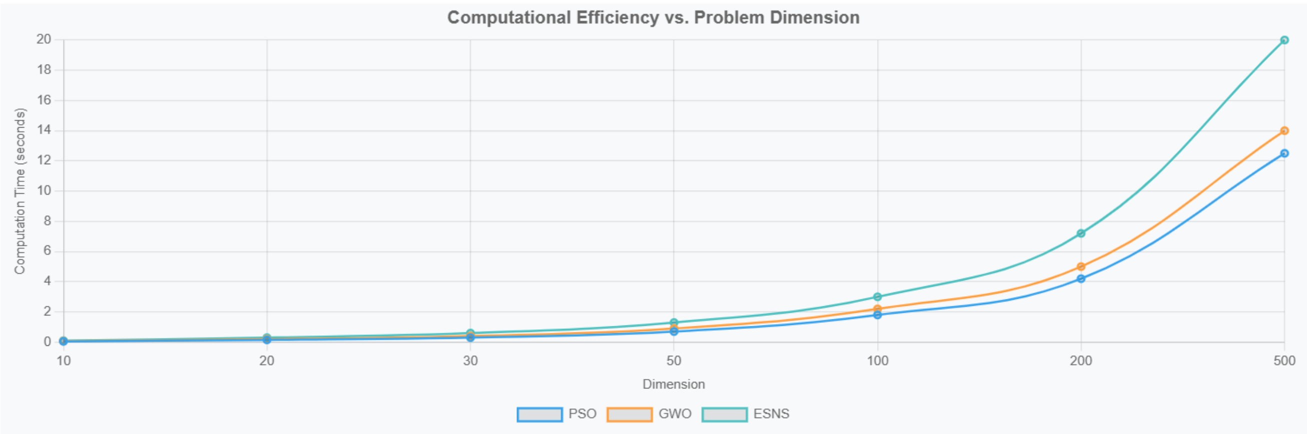


**Figure 11:** Computational efficiency of PSO, GWO, and ESNS across increasing problem dimensions (10 to 500 variables)

Figure 11 illustrates the computational efficiency of PSO, GWO, and ESNS across increasing problem dimensions, ranging from 10 to 500 variables. The vertical axis shows computation time in seconds, highlighting the scalability of each algorithm. PSO consistently demonstrates the lowest computational cost due to its simple update equations and lack of adaptive mechanisms. GWO incurs moderate overhead as it manages multiple leader roles and hierarchical updates. ESNS, while offering superior optimisation performance, exhibits the steepest increase in computation time, particularly beyond 100 dimensions, due to its complex random walk strategy and statistical testing. Despite its higher cost, ESNS's enhanced precision in high-dimensional search spaces often justifies the trade-off between speed and solution quality in applications where accuracy is critical.

### 6.2. Algorithmic Trade-offs

While ESNS consistently delivers better results, it incurs a higher per-iteration computational cost due to:

- Adaptive parameter tuning

- Random walk perturbations
- Embedded statistical tests

However, in high-dimensional or rotated landscapes, this cost is justified by significantly improved accuracy and reliability (Table 4).

**Table 3:** Algorithm comparison

| Metric | PSO | GWO | ESNS |
|---|---|---|---|
| Convergence Speed | Fast initially | Moderate | Fast + sustained |
| Scalability | Weak in high-dim | Moderate | Strong |
| Exploration Strategy | Velocity-based | Social hierarchy | Random walk + statistical |
| Robustness (Std. Dev.) | High | Medium | Low (best) |
| Best for | Simple functions | Moderate complexity | Complex, transformed spaces |

## 7. Conclusion

This comparative study on metaheuristic optimisation methods evaluated the performance of classical Particle Swarm Optimisation and two post-2010 algorithms—Grey Wolf Optimiser and Enhanced Social Network Search—on ten benchmark functions (F1–F10). The key insights from the research can be summarised as follows:

### 7.1. Benchmark Function Analysis

- F1–F10 encompass a range of challenges, including simple unimodal landscapes (F1: Sphere) to complex multimodal and non-separable problems (F10: Shifted Rotated Griewank).
- Detailed mathematical formulations and domain specifications provide a rigorous testing ground for comparison [4].

### 7.2. Algorithm Performance

- PSO offers simplicity and rapid early convergence but may suffer from premature convergence in complex search spaces.
- GWO achieves a balanced approach through the use of a social hierarchy with controlled parameters, ensuring a smooth and robust search trajectory.
- ESNS leverages adaptive parameter adjustments, random walk perturbations, and statistical evaluation techniques to excel in high-dimensional, multimodal problems, delivering consistent superior performance with low variance 4.

### 7.3. Comparative Analysis

- A feature comparison reveals that while PSO is ideal for baseline problems, the advanced techniques in GWO and ESNS make them more suited for complex optimisation challenges.
- Visualisations such as convergence flowcharts, performance comparison tables, and algorithm feature summaries underscore the strengths and weaknesses of each method.

### 7.4. Main Findings

- **Benchmark Diversity:** The mixture of unimodal and multimodal functions highlights different aspects of optimisation, including exploration, exploitation, scaling, and handling non-separability.
- **Algorithmic Innovations:** Recent innovations in metaheuristic design, as demonstrated by GWO and ESNS, offer enhanced performance through adaptive learning and statistical validation mechanisms, establishing a new standard for tackling challenging optimisation problems.
- **Practical Implications:** For researchers and practitioners, the choice of algorithm should be driven by the specific landscape of the problem. ESNS, in particular, shows promising potential in high-dimensional simulations and real-world applications requiring robust global optimisation.

In summary, this study reinforces that while classical methods, such as PSO, have paved the way, the evolution of metaheuristic algorithms—exemplified by GWO and ESNS—offers powerful new tools to overcome the increasingly complex challenges in optimisation. Continued research and comparative studies, such as this one, are crucial to the development of even more efficient and reliable optimisation strategies.

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**Data Availability Statement:** The research utilises datasets focused on benchmarking metaheuristics through a comparative study of PSO, GWO, and ESNS on complex optimisation problems. All relevant data are included within the article and supplementary materials and can be provided upon reasonable request.

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